Polynomiality of the structure coefficients of some combinatorial algebras related to the symmetric group

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Abstract: The algebra of the symmetric group $\mathbb{C}[\mathcal{S}_n]$ is the algebra of linear combinations of permutations of size n. A basis of the center of $\mathbb{C}[\mathcal{S}_n]$ is given by $\mathbf{C}_\lambda$, which are the sums of permutations with cycle-type λ. The structure coefficients $c_{\lambda\delta}^{\gamma}$ describe the product in the center of $\mathbb{C}[\mathcal{S}_n]$:

$$\mathbf{C}_\lambda \mathbf{C}_\delta = \sum_{\gamma} c_{\lambda\delta}^{\gamma} \mathbf{C}_\gamma$$

In 1958, Farahat and Higman proved the polynomiality of the coefficients $c_{\lambda\delta}^{\gamma}$ in n when λ, δ and γ are fixed partitions, completed with parts equal to 1 to get partitions of n. In 1999, Ivanov and Kerov gave a new (combinatorial) proof of this result through the introduction of partial permutations. In this talk, we will first recall these results. Second, we will give an analogous statement arising in the context of Hecke algebra of the pair $(\mathcal{S}_{2n}, \mathcal{H}_n)$, where $\mathcal{H}_n$ is the hyperoctahedral subgroup of $\mathcal{S}_{2n}$. If time permits, we will show that the Ivanov and Kerov's construction may be extended to include more interesting algebras.